INDIAN MARITIME UNIVERSITY

(A Central University, Govt. Of India)

End Semester Examination December 2018

B. Tech. (Marine Engineering)

Semester - I

Mathematics - I (UG11T3102)

Date: 29.12.2018 Max Marks: 100

Time: 3 Hrs. Pass Marks: 50

PART – A **(3 x10 = 30)**

Compulsory Questions: (The symbols have their usual meanings.)

1.

(a) Find the nth derivative of $y = x \log \frac{x-1}{x+1}$.

(b) If $u = f(x, y)$ and $= r\cos\theta$, $y = r\sin\theta$, prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

(c) If $u^3 + v^3 = x + y$ and $u^2 + v^2 = x^3 + y^3$, show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2}\frac{y^2 - x^2}{uv(u-v)}$.

(d) Find the radius of curvature at any point on the curve

$$x = a(\theta + \sin\theta), \quad y = a(1 - \cos\theta).$$

(e) Express the integral $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma function.

(f) Evaluate the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates.

(g) Find the unit normal vector to the surface $xy^2z = 3x + z^2$ at the point $(-1, -1, 2)$.

(h) Using Cayley Hamilton theorem find the A^{-1} of matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

(i) Show that shortest distance between two points in a plane is a straight line.

(j) Graphically find the maximum value of $Z = x_1 + x_2$ subject to the constraints $x_1 + 2x_2 \geq 2$, $x_1 \leq 3$, $x_2 \leq 4$, $x_1,\ x_2 \geq 0$.

PART – B **(14 x 5 = 70)**

Answer any <u>FIVE</u> of the following questions

2(a) If $y = (\sin^{-1} x)^2$, prove that

(i) $(1 - x^2)y_2 - xy_1 = 2$

(ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$. [3+4]

(b) Find the asymptotes of the curve
$$4x^3 + 2x^2 - 3xy^2 - y^3 - 1 - xy - y^2 = 0.$$ [7]

3(a) If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$, prove that
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$ [7]

(b) Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$. [7]

4(a) Evaluate the double integral $\iint e^{2x+3y} dxdy$ over the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$. [7]

(b) Evaluate triple integral $\int_0^1 \int_{y^2}^1 \int_0^{1-x} xdz\, dx\, dy$. [7]

5(a) Find the directional derivative of $\emptyset = x^2y^2z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve
$x = e^t,\ \ y = \sin 2t + 1,\ \ z = 1 - \cos t$ at $t = 0$. [7]

(b) Show that the vector field $\vec{F} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$ is irrotational. Find a scalar potential function $\emptyset$ such that $\vec{F} = \nabla\emptyset$. [7]

6(a) Discuss the consistency of the following system of equations and solve it if consistent.
$$2x - y + 3z = 4,$$
$$x + y - 3z = -1,$$
$$5x - y + 3z = 7.$$ [7]

(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

7(a) Let $F(z) = u(x, y) + iv(x, y)$ be an analytic function of z. If u $= x^3 - 3xy^2 + 3x^2 - 3y^2$ then find the v and express $f(z)$ in terms of z. [7]

(b) Evaluate the integral $\oint_C \frac{e^{-z}}{(z-1)(z-2)^2} dz$, where C is the circle $|z| = 3$. [7]

8(a) Using simplex method solve the following LPP
Maximize $Z = 5x_1 + 3x_2$
subject to $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1,\ x_2 \geq 0$. [7]

8(b) Find the curve on which functional $\int_0^2 (x + y')y'dx$ with $y(0) = 0$ and $y(2) = 1$ can be extremized. [7]

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